Leibniz's new kind of Logic

James Cussens

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The promise of 'logical' probability

- There are a number of theories of what probability is: subjective (e.g. de Finetti), objective (e.g. von Mises), propensity (e.g. Popper).
- The logical interpretation of probability is particularly appealing in situations of radical uncertainty.
- The idea is that given some statements about the world (perhaps expressed in some suitable formal language), the probability of any uncertain event is determined and can be computed, conditional on those statements.
- This is a very optimistic (one might say Panglossian) view—a partial solution to the problem of induction, no less.

Inductive logic vs logics of probability

- One must differentiate a logic that infers probabilities from a description of some state-of-affairs from one that merely infers probabilities (or perhaps bounds on probabilities) from other probabilities.
- The latter approach is entirely deductive.
- This latter approach can be found in the AI literature, where, for example, we have probability distributions over 'possible worlds' (i.e. models).
- However, there is some work in AI that formalises an inductive approach: "The method is based on the principle of indifference: it treats all of the worlds the agent considers possible as being equally likely." [Bacchus et al., 1996]

Outline

Deduction is not enough

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- As noted by Hacking [Hacking, 1975, p. 135], Leibniz did not view deduction as sufficient for reasoning.
- Probabilistic reasoning was a key part of his *universal characteristic*.

Probability in the Universal Characteristic

Our Characteristic, however, will reduce all questions to numbers, and thus present a sort of statics by virtue of which rational evidence may be weighed. Besides, since probabilities lie at the basis of estimation and proof, we can consequently always estimate which event under given circumstances can be expected with the highest probability. [Leibniz, 1677]

Leibniz and probability

So . . .

- how should we do probabilistic reasoning?

Outline

Legal reasoning and partial entailment

Jurisconsults in treating the proofs, presumptions, conjectures and indices have said a number of good things on the subject (of degrees of assent) and have entered into considerable detail. . . . there are many degrees of conjecture and indices. . . . the entire form of juridical procedures is in fact nothing but a species of Logic applied to questions of law. Physicians also have a number of degrees and differences in their signs and indications which may be seen among them. [Leibniz, 1705, bk. IV, ch. XVI]

Connecting chance and judgement

The Mathematicians of our day have begun to estimate chances in connection with gambling games.. . . I have more than once said that we should have a new kind of Logic which would treat of degrees of probability, . . . [Aristotle] did not take the trouble to give us a necessary balance to weigh probabilities and to form solid judgements accordingly. It would be well for future investigators of this matter to pursue the examination of games of chance; . . . [Leibniz, 1705, bk. IV, ch. XVI]

Outline

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Even if it is only a question of probabilities we can always determine what is most probable on the given premises. True this part of useful logic is not yet established anywheres, but it is put to wonderful uses in practice when there are hypotheses, indications, and conjectures involved in ascertaining degrees of probability among a number of reasons appearing on one side or another of some important deliberation.

Leibniz's new kind of Logic

Thus when we lack sufficient data to demonstrate a certitude, the matter yielding only probability, we can always give demonstrations at least concerning the probability itself. I do not speak here of that probability of the Casuists, based on the number and reputation of scholastic doctors, but of that probability drawn from the nature of things in proportion to what we know of them and which we may call likelihood. [Leibniz, 1680]

Inductive Logic

- This is the birth of inductive logic
- Keynes: *A Treatise on Probability* 1921
- Carnap: *On the application of inductive logic* 1947, *Logical Foundations of Probability* 1950, *The continuum of inductive methods* 1952
- Popper not a fan . . .

Modern probabilistic reasoning is deductive

$$P(h|e) = \frac{P(h)P(e|h)}{P(e)} \tag{1}$$

As Franklin notes, when, in 1703, Bernoulli asked for legal examples where posterior probabilities could be applied, Leibniz could supply nothing. Leibniz thus represents "not only the coming together of legal and mathematical probability, but also their divergence" [Franklin, 2001, p.365]

Outline

aequalibus aequalia

The foundation on which [De Witt computed annuities] goes back to prosthaphaeresis, that is, to taking an arithmetical mean among several equally acceptable hypotheses, and our peasants have long used it in doing their natural mathematics. . . . [They use] the axiom of aequalibus aequalia, equal hypotheses must have equal weight. [Leibniz, 1705, bk. IV, ch. XVI]

Fairness and probability in *De incerti aestimatione* (1678)

> *A game is fair if there is the same proportion of hope to fear on either side. [de Melo and Cussens, 2004]*
>
> *Axiom. If players do similar things in such a way that no distinction can be drawn between them, with the sole exception of the outcome, there is the same proportion of hope to fear. [de Melo and Cussens, 2004]*

Note: the players need not be using a symmetric die.

> *Probability is the degree of possibility. The hope is the probability of having. The fear is the probability of losing. The estimated value of a thing is as high as each one's claim to it. [de Melo and Cussens, 2004]*

Feasibility

L'art de conjecturer est fondée sur ce qui est plus ou moins facile, ou bien plus ou moins faisable, car le latin facilis derivé a faciendo veut dire faisable mot à mot: par exemple, avec deux dés, il est aussi faisable de jetter douze points, que d'en jetter onze, car l'un est l'autre ne se peut faire que d'une seule maniere; mais il est trois fois plus faisable d'en jetter sept, parce que cela se peut fair en jettant 6 et 1, 5 et 2, et 4 et 3; et une combinaison icy est aussi faisable que l'autre. [Leibniz, 0 61]

Probabilities *a posteriori*

One may still estimate likelihoods [vraisemblances] a posteriori, by experience; to which one must have recourse in default of a priori reasons. For example, it is equally likely that a child should be born a boy or a girl, because the number of boys and girls is very nearly equal all over the world. One can say that what happens more or less often is more or less feasible in the present state of things, putting together all considerations that must concur in the production of a fact. [Leibniz, 0 61], translation is [Hacking, 1975, p.128].

Outline

Leibnizian determinism

And even this effect of small things causes those who do not consider things correctly to imagine some things happen accidentally and are not determined by destiny, for this distinction arises not in the facts but in our understanding, just as one who does not comprehend the large number of small things belonging to every particular effect and does not recognize any cause he does not see, will then imagine that aces turn up in dice simply by chance. [Leibniz, nown]

Sufficient reason?

There must always be some foundation of the connection of terms in a proposition, which must be found in their notions. This is my great principle, with which I believe all philosophers must agree, and of which one of the corollaries is this vulgar axiom, that nothing happens without a reason . . . though often this reason inclines without necessitating. quoted in [Russell, 1937, 32–33] (1686)

Sufficient reason and freedom

There is always a reason or greater inclination behind what we in fact choose to do; our choice may depend not only on argument, good or bad, but also on passions, habits dispositions of the organs of thought, external impressions, greater or less attention, etc. This does not destroy freedom although it inclines it. [Leibniz, ated]

Outline

Hacking's view

Leibniz had two justifications of expectation, one based on insufficient reason, and one based on physical equipossibility. He does not actually say there are two distinguishable ideas of probability but he does present two differentiated arguments. [Hacking, 1975]

Daseinstreben

. . . all possible things, . . . tend by equal right towards existence, according to their degree of essence or reality, or according to the degree of perfection they contain, for perfection is nothing else than quality of essence. [Leibniz, 1697]

Wilson's view

As a theory of the origination of things the Da-seinstreben presents an all-or-nothing proposition: that fully determinate possible world with the most perfection achieves existence, and all its competitors fail. Certainly to say that possible world W has twice the perfection of possible world W′ is not to say that the former may be expected to occur twice as often as the latter! Neither, however, is it to say that it has twice the chance of occurring. And it is not clear what sense could be ascribed, within Leibniz's system, to the claim that one world is more makeable than another. [Wilson, 1971]

Contingency

All contingent propositions have reasons for being as they are rather than otherwise, or (what is the same thing) they have a priori proofs of their truth, which render them certain, and show that the connection of subject and predicate in these propositions has its foundation int the nature of the one and the other; but they do not have demonstrations of necessity, since these reasons are only founded on the principle of contingency, or of the existence of things, i.e. on what is or appears the best among several equally possible things [Leibniz, 1686b]

Cussens's view

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- So *simplicity* and *fecundity* will be strongly represented in the actual (= best possible) world.
- This is a good inductive bias.

Leibniz on the best possible world

Thus we may say that in whatever manner God might have created the world, it would always have been regular and in a certain order. God, however, has chosen the most perfect, that it to say the one which is at the same time the simplest in hypotheses and the richest in phenomena . . . [Leibniz, 1686a, 6]


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